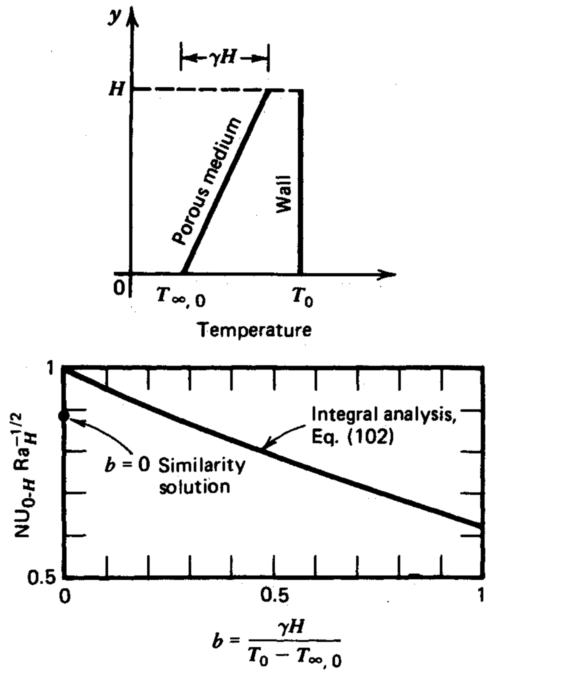

Fig. 1 Heat transfer from a vertical isothermal wall to a linearly stratified fluid-saturated porous medium.

in this notation reads

$$Nu_{0-H} = 0.888\, Ra_H^{1/2} \tag{1}$$

This shows that the integral solution overestimates by 13% the overall heat transfer rate when $b = 0$. In other words, this degree of accuracy of the integral solution was noted by me in 1984: it is not Lai et al's discovery.

The new and simpler solution is based on the observation that Eq. (1) holds approximately even when stratification is present, provided the Nusselt and Rayleigh groups are based on the *H-averaged* temperature difference

$$\overline{\Delta T} = \Delta T([1 - b/2]) \tag{2}$$

Therefore, in place of Eq. (1), we write approximately

$$\frac{\overline{q''}H}{k\overline{\Delta T}} \cong 0.888 \left(\frac{g\beta K H \overline{\Delta T}}{\alpha\nu}\right)^{1/2} \tag{3}$$

and, after reverting to the ΔT-based notation (Nu_{0-H}, Ra_H) defined earlier, we obtain approximately

$$Nu_{0-H} \cong 0.888([1 - b/2])^{3/2}\, Ra_H^{1/2} \tag{4}$$

On the attached figure, Eq. (4) would generate a curve that falls under the solid curve of the integral solution

b	0	0.25	0.5	0.75	1
$Nu_{0-H}Ra_H^{-1/2}$	0.888	0.73	0.58	0.44	0.31

However, Eq. (4) has the advantage that it is exact in the limit $b \rightarrow 0$. It would be interesting to compare Eq. (4) with the $Nu_{0-H}Ra_H^{-1/2} = f(b)$ curve that would result from the local nonsimilarity analysis employed by Lai et al. (note that Lai et al. did not report the overall Nusselt number).

References

[1]Bejan, A., *Convection Heat Transfer*, Wiley, New York, 1984, pp. 367–371.

[2]Lai, F. C., Pop, I., and Kulacki, F. A., "Natural Convection from Isothermal Plates Embedded in Thermally Stratified Porous Media," *Journal of Thermophysics and Heat Transfer*, Vol. 4, No. 4, 1990, pp. 533–535.

[3]Cheng, P., and Minkowycz, W. J., "Free Convection about a Vertical Flat Plate Embedded in a Saturated Porous Medium with Application to Heat Transfer from a Dike," *Journal of Geophysical Research*, Vol. 82, 1977, pp. 2040–2044.

Reply by the Authors to A. Bejan

F. C. Lai* and F. A. Kulacki*
Colorado State University,
Fort Collins, Colorado 80523

We thank Dr. Bejan for his comments on our paper, and we offer the following in reply.

As stated in the introduction, the focus of the paper[1] is to extend the earlier study of Nakayama and Koyama[2] to include a more realistic temperature stratification that may be encountered in applications. Our presentation of the history of the problem may be incomplete, and we appreciate Dr. Bejan's reference to his earlier study.[3] However, we have given his work credit in the comparison of our results with his integral solution. More importantly, we do not claim the discovery of the discrepancy found in the solutions we present. It is well known that the accuracy of an integral solution is largely dependent on the approximate solution form and that accuracy can be greatly improved if a better functional form can be adopted. This is exactly the conclusion of our paper.

Although our results did not include the overall heat transfer rate, the calculation of this value is straightforward. Following the notation used by Bejan, we have:

$$Q = \int_0^H - k\left.\frac{\partial T}{\partial x}\right|_{x=0} dy$$

$$= k(T_0 - T_{\infty,0})Ra_H^{1/2}b^{-1/2}\int_0^b -\Theta'(\xi, 0)\xi^{1/2}\, d\xi \tag{1}$$

The result is most informative if the overall heat transfer rates thus obtained are compared to those of a plate in an isothermal environment; that is, no thermal stratification. The

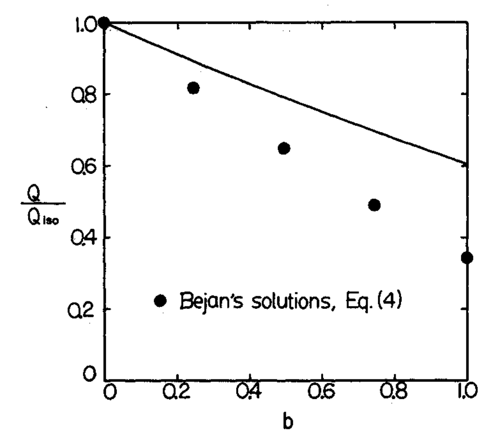

Fig. 1 Ratio of overall heat transfer rates, Eq. (3), as a function of the stratification parameter, *b*

Received Oct. 4, 1991; revision received Oct. 28, 1991; accepted for publication Oct. 30, 1991.

*Department of Mechanical Engineering.

latter is given by[4]

$$Q_{iso} = k(T_0 - T_{\infty,0})Ra_H^{1/2}[-2\Theta'(0)] \tag{2}$$

Thus, the ratio of overall heat transfer rates is

$$\frac{Q}{Q_{iso}} = \frac{b^{1/2}\int_0^b -\Theta'(\xi, 0)\xi^{-1/2}\,d\xi}{0.888} \tag{3}$$

The integration in Eq. (3) can be evaluated numerically, with the $\Theta'(\xi, 0)$ taken from our paper.[1] The result is plotted in Fig. 1 in terms of the parameter b. The new solutions proposed by Bejan are also included for comparison. His results underestimate those given by Eq. (3). This indicates that the assumptions employed by him in deriving his solution may not be theoretically correct, especially when b varies over a large range.

References

[1]Lai, F. C., Pop, I., and Kulacki, F. A., "Natural Convection from Isothermal Plates Embedded in Thermally Stratified Porous Media," *Journal of Thermophysics and Heat Transfer,* Vol. 4, No. 4, 1990, pp. 533–535.

[2]Nakayama, A., and Koyama, H., "Effects of Thermal Stratification on Free Convection with a Porous Medium," *Journal of Thermophysics and Heat Transfer,* Vol. 1, No. 3, 1987, pp. 282–285.

[3]Bejan, A., *Convection Heat Transfer,* Wiley, New York, 1984, pp. 367–371.

[4]Cheng, P., and Minkowycz, W. J., "Free Convection About a Vertical Flat Plate Embedded in a Saturated Porous Medium with Application to Heat Transfer from a Dike," *Journal of Geophysical Research,* Vol. 82, No. 14, 1977, pp. 2040–2044.